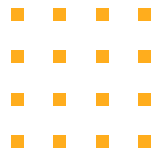
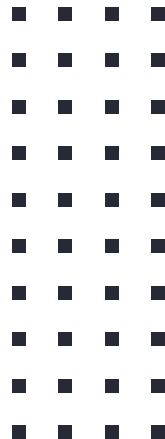




Class 3: Number Sense (Continued)



Homework Take-up – Problem 1

Find the value of the following using BEDMAS:

(a) $4 - 5 \times 1$

(b) $9 + 3(5 - 3)$

(c) $14 \div (2 + 5)$

$$\begin{aligned} \text{(a)} \quad 4 - 5 \times 1 &= 4 - 5 \\ &= -1 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 9 + 3(5 - 3) &= 9 + 3 \times 2 \\ &= 9 + 6 \\ &= 15 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad 14 \div (2 + 5) &= 14 \div 7 \\ &= 2 \end{aligned}$$

$$n^{\text{th}} \text{ term} = \text{first term} + (n - 1) \times d$$

$$\text{Sum} = (\text{first term} + \text{last term}) \times (\text{number of terms}) \div 2$$

Homework Take-up – Problem 2

Find the 28th term of the following sequence: 2, 6, 10, 14, 18, ...

$$\begin{aligned} 28^{\text{th}} \text{ term} &= 2 + (28 - 1) \times 4 \\ &= 2 + 27 \times 4 \\ &= 2 + 108 \\ &= 110 \end{aligned}$$

$$n^{\text{th}} \text{ term} = \text{first term} + (n - 1) \times d$$

$$\text{Sum} = (\text{first term} + \text{last term}) \times (\text{number of terms}) \div 2$$

Homework Take-up – Problem 3

Find the sum of the following sequence: 3, 8, 13, 18, 23, 28, 33.

$$\begin{aligned}\text{Sum} &= (3 + 33) \times 7 \div 2 \\ &= 36 \times 7 \div 2 \\ &= 252 \div 2 \\ &= 126\end{aligned}$$

Warm-up Problem 1

A palindrome is a positive integer that is the same when read forwards or backwards. For example, 545 and 1331 are both palindromes. What is the difference between the smallest three-digit palindrome and the largest three-digit palindrome? (Source: Gauss 7)

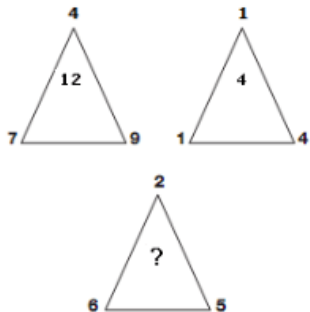
Smallest three-digit palindrome = 101

Largest three-digit palindrome = 999

Difference = $999 - 101$
= 898

Warm-up Problem 2

Which number replaces the question mark? (Find the pattern)



Pattern: Number inside triangle = Sum of bottom numbers – Top number

$$? = 6 + 5 - 2$$

$$= 11 - 2$$

$$= 9$$

Problem from Previous Day (Q1)

Today is Tuesday. Which day of the week will it be 80 days later, not including today?

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
R5	R6	R0	R1	R2	R3	R4

Each week cycles with 7 days.

Since today is Tuesday, the remainder when divided by 7 should be 0.

Then, Wednesday has remainder 1, Thursday has remainder 2, etc.

$$80 \div 7 = 11 \text{ R}3$$

Friday has remainder 3.

\therefore It will be a Friday 80 days later.

Problem from Previous Day (Q2)

A string of beads are arranged in the following pattern: 5 red, 2 green, 3 blue, 5 red, 2 green, 3 blue, and so on. What colour is the 146th bead?

$$5 + 2 + 3 = 10 \text{ beads per cycle}$$

$$146 \div 10 = 14 \text{ R}6$$

The 146th bead is the 6th bead in the 10th cycle.



\therefore The 146th bead is green.

Exponents

Exponents are expressed in the form a^b . It is read “a to the power of b”
a is the **base**, and b is the **exponent**.

$$a^b = \underbrace{a \times a \times a \times \cdots}_{b \text{ number of } a\text{'s}}$$

For example,

$$3^2 = 3 \times 3 = 9$$

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

Exercise

Calculate the following values:

(a) 2^5

(b) 6^2

(c) 4^3

(d) 3^1

(a) $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

(b) $6^2 = 6 \times 6 = 36$

(c) $4^3 = 4 \times 4 \times 4 = 64$

(d) $3^1 = 3$

Prime Numbers & Composite Numbers

Prime numbers are whole numbers that are only divisible by **1 and itself**. Composite numbers are whole numbers that have more than 2 factors.

For example, 2, 3, and 5 are examples of **prime numbers**, but 4, 6, 8, 9 are examples of **composite numbers**.

Note that 1 is neither prime nor composite – it is a **special number**.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

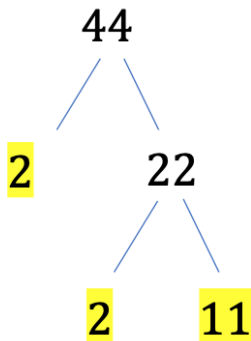
Prime Factorization

To **prime factorize** a number means to express it as a product of multiple prime numbers.

Here are some examples of prime factorizations:

$$14 = 2 \times 7$$

$$12 = 2^2 \times 3$$



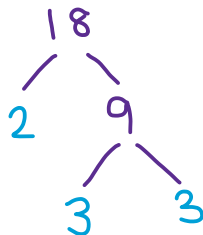
A tree diagram can be useful in prime factorization. Once all the numbers at the bottom are prime numbers, the prime factorization is complete.

$$44 = 2^2 \times 11$$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

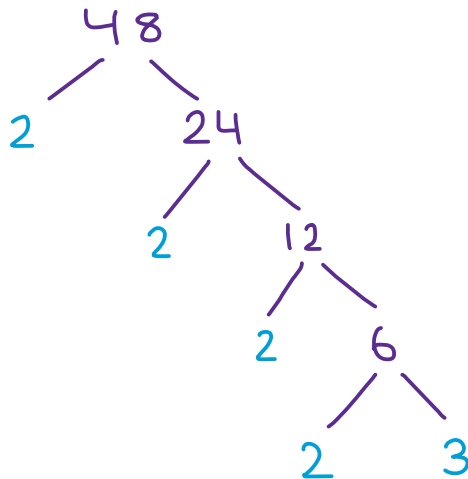
Examples

Prime factorize 18:



$$18 = 2 \times 3^2$$

Prime factorize 48:



$$48 = 2^4 \times 3$$

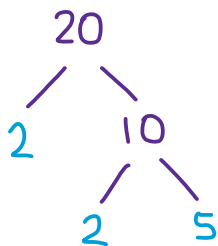
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Exercise

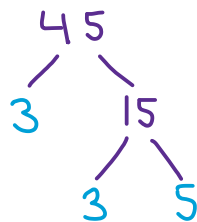
Prime factorize the following values:

(a) 20

(b) 45



$$20 = 2^2 \times 5$$



$$45 = 3^2 \times 5$$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Lowest Common Multiples (LCM)

A **common multiple** of 2 numbers is a number that is divisible by both of these numbers.

For example, a common multiple of 2 and 5 is 10, since 10 is divisible by both 2 and 5. Another common multiple would be 20.

The **lowest common multiple** of 2 numbers is the smallest of the common multiples. For example, 10 and 20 are both common multiples of 2 and 5, but 10 is the **lowest common multiple** of 2 and 5.

How to find the LCM:

1. Prime factorize each number. Use exponents.
2. Take the highest exponent of every factor from each number.

Example: Find the LCM of 4 and 10.

$$4 = 2^2$$

$$10 = 2 \times 5$$

$$\begin{aligned}\text{lcm}(4, 10) &= 2^2 \times 5 \\ &= 4 \times 5 \\ &= 20\end{aligned}$$

Example: Find the LCM of 12 and 15.

$$12 = 2^2 \times 3$$

$$15 = 3 \times 5$$

$$\begin{aligned}\text{lcm}(12, 15) &= 2^2 \times 3 \times 5 \\ &= 4 \times 3 \times 5 \\ &= 60\end{aligned}$$

1. Prime factorize each number. Use exponents.
2. Take the highest exponent of every factor from each number.

Exercise

Find the LCM of the following pairs of numbers:

(a) $\text{lcm}(4, 6)$

(b) $\text{lcm}(5, 6)$

(c) $\text{lcm}(2, 18)$

(a) $4 = 2^2$

$6 = 2 \times 3$

$$\begin{aligned}\text{lcm}(4, 6) &= 2^2 \times 3 \\ &= 4 \times 3 \\ &= 12\end{aligned}$$

(b) $5 = 5$

$6 = 2 \times 3$

$$\begin{aligned}\text{lcm}(5, 6) &= 2 \times 3 \times 5 \\ &= 30\end{aligned}$$

(c) $2 = 2$

$18 = 2 \times 3^2$

$$\begin{aligned}\text{lcm}(2, 18) &= 2 \times 3^2 \\ &= 18\end{aligned}$$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1. Prime factorize each number. Use exponents.
2. Take the highest exponent of every factor from each number.

Problem

A subway departs from the station every 6 minutes. A train departs from another station every 8 minutes. If they both depart at 12:00 PM, what is the next time they will both depart at the same time?

Since the problem is asking for the **next time** they both depart at the same time, we want to find the LCM of 6 and 8.

$$6 = 2 \times 3$$

$$8 = 2^3$$

$$\begin{aligned} \text{lcm}(6, 8) &= 2^3 \times 3 \\ &= 8 \times 3 \\ &= 24 \end{aligned}$$

They will both depart at the same time again in 24 minutes.

Since it is 12:00 PM right now, they will depart together at **12:24 PM**.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1. Prime factorize each number. Use exponents.
2. Take the highest exponent of every factor from each number.

Problem

There are two sand timers, A and B. Timer A takes exactly 30 minutes for the sand to run out, and Timer B takes exactly 50 minutes for the sand to run out. Both timers are started at the same time and are immediately flipped over and reset once it runs out of sand. After how many minutes will both timers run out of sand at the same time for the first time?

We want to find the LCM of 30 and 50.

$$30 = 2 \times 3 \times 5$$

$$50 = 2 \times 5^2$$

$$\begin{aligned}\text{lcm}(30, 50) &= 2 \times 3 \times 5^2 \\ &= 6 \times 25 \\ &= 150\end{aligned}$$



Both timers will run out of sand at the same time after **150 minutes**.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100